# Thema

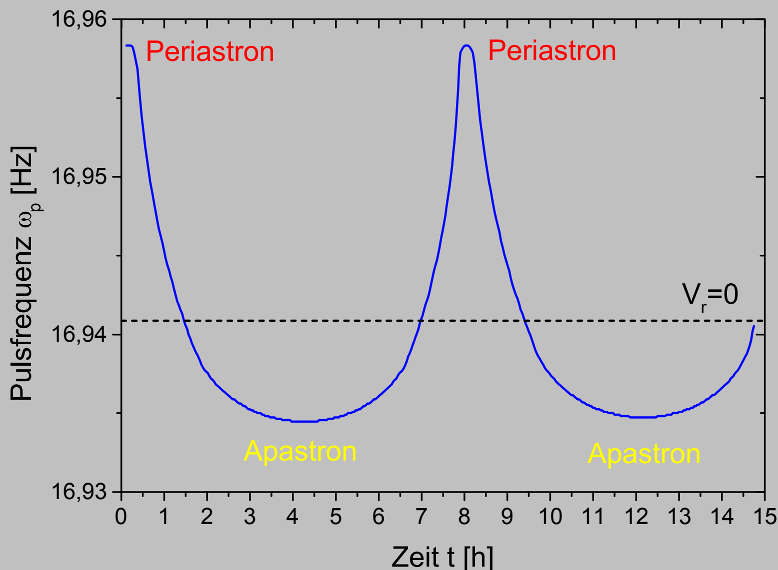
Laserinterferometrie,
eine
potentielle Meßmethode
zur
Gravitationswellendetektion ?

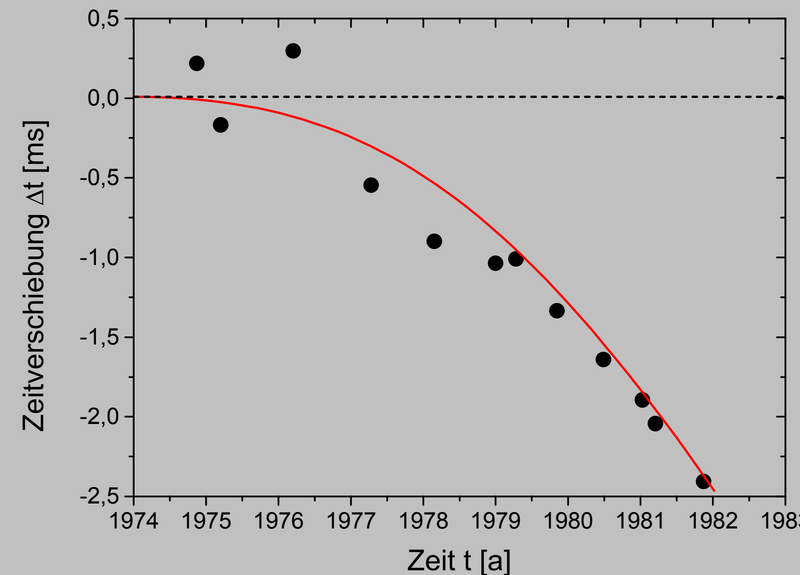
# Gliederung

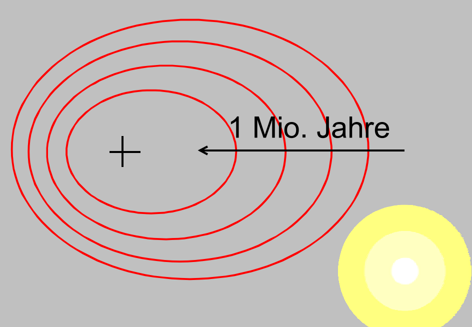
- Einführung
- Gravitationswellen
- Prinzip eines Laserinterferometers
- Rauschquellen, Rauschunterdrückung
- Betrieb
- Methoden zur Empfindlichkeitssteigerung
- Vergleich zu resonanten Massenantennen
- Weltraumgebundene Interferometer
- Weltweite Aktivitäten und Erfolgsaussichten
- Zusammenfasung

# Nobel-Preis für Physik 1993 (Hulse und Taylor)

## Entdeckung des Radio-Pulsars PSR 1913+16 (1974):



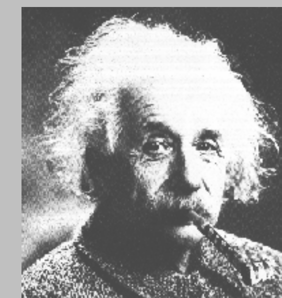




$\Rightarrow$ indirekter Nachweis für Gravitationswellen

# Einsteins Feldgleichungen

Ausgangspunkt: $R_{\mu\nu} = -\frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}\right)$



(Einstein 1915)

$R_{\mu\nu} = R_{\mu\nu}(g_{\mu\nu})$ Krümmungstensor

$T_{\mu\nu}$ Energie-Impuls-Tensor

$g_{\mu\nu}$ Metrik-Tensor $\Longrightarrow$ Raumgeometie

$\Longrightarrow$ Abstände $ds^2 = g_{\mu\nu} \cdot x^\mu \cdot x^\nu$

z.B.:

$$g_{\mu\nu} = \delta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},\ ds^2 = dx^2 + dy^2 + dz^2$$

(Eukidische Metrik)

$$g_{\mu\nu} = \eta_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix},\ ds^2 = c^2 t^2 - dx^2 - dy^2 - dz^2$$

(Minkowski-Metrik)

# Einsteins Feldgleichungen

Für schwache Felder: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ mit $|h_{\mu\nu}| \ll 1$

zusammen mit: $$\frac{\partial h^{\mu}{}_{\nu}}{\partial x^{\mu}} - \frac{1}{2}\frac{\partial h^{\mu}{}_{\mu}}{\partial x^{\nu}} = 0$$ (Einstein-Eichung)

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) h_{\mu\nu} = -\frac{16\pi G}{c^4}\cdot S_{\mu\nu} \quad \text{mit} \quad S_{\mu\nu} = T_{\mu\nu} - \frac{T}{2}\eta_{\mu\nu}$$ (Wellengleichung)

Lösung in Analogie zur Elektrodynamik:

$$h_{\mu\nu}(\vec{r},t) = -\frac{4G}{c^4}\int d^3r' \, \frac{S_{\mu\nu}(\vec{r}\,', t - |\vec{r} - \vec{r}\,'| / c)}{|\vec{r} - \vec{r}\,'|}$$ (Retardierte Potentiale)

# Testmassen im Feld einer ebenen Welle

Für ebene Wellen :

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta\right) h_{\mu\nu} = 0$$

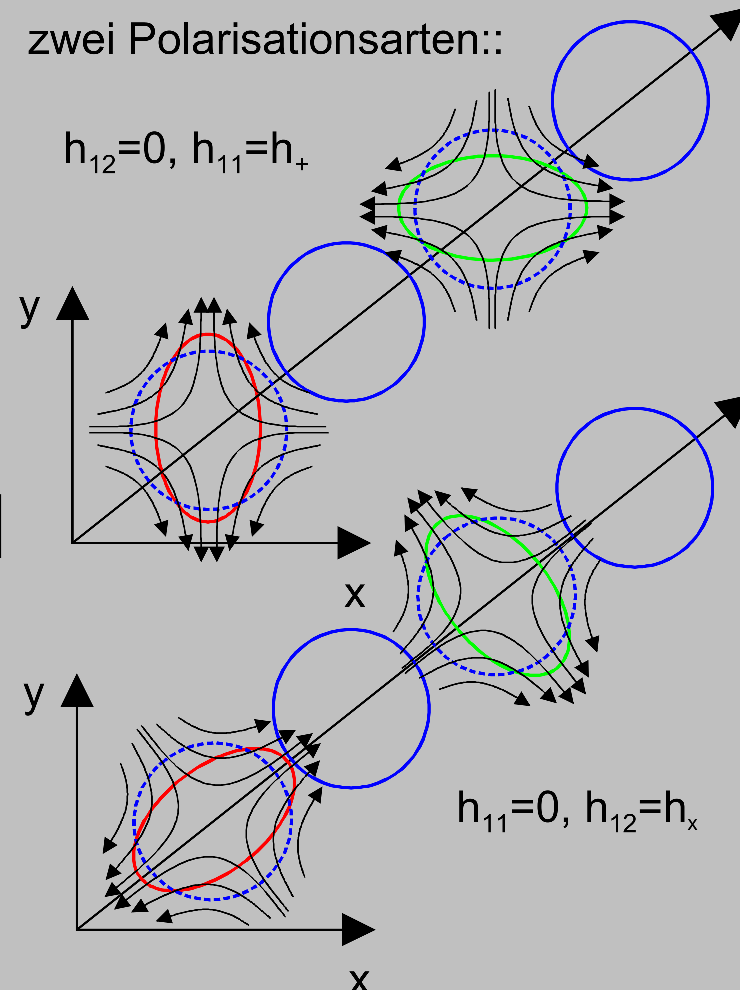
mit Lösung:

$$h_{\mu\nu}(z,t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{12} & 0 \\ 0 & h_{12} & -h_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \exp[ik(z-ct)]$$

Energiestromdichte:

$$\Phi_{GW} = \frac{c^3}{8\pi G} \cdot \omega^2 \cdot h^2$$

zwei Polarisationsarten::

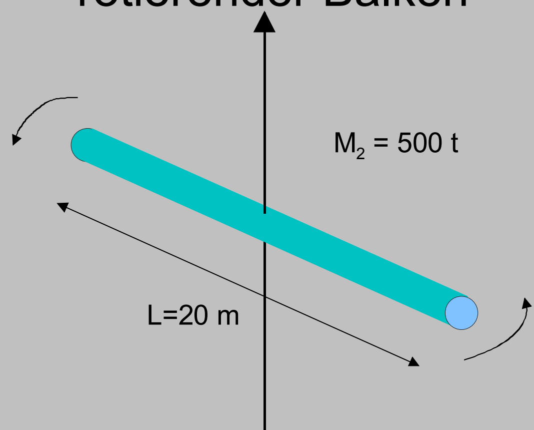


# Testmassen im Feld einer ebenen Welle

Quadrupolstrahlung einer rotierenden Massenverteilung:

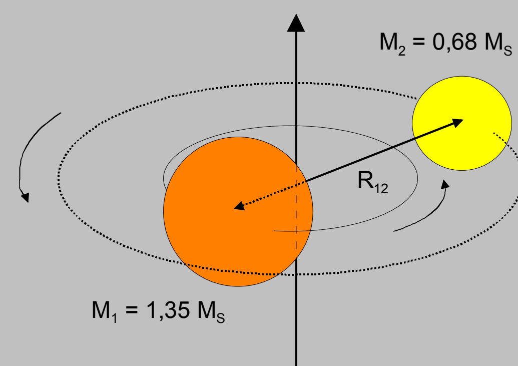
$$P = \frac{2G\omega^6}{5c^5}\left(\sum_{i,j=1}^{3}\left|Q^{ij}\right|^2 - \frac{1}{3}\left|\sum_{i=1}^{3}Q^{ii}\right|^2\right) = \frac{32G\omega^6}{5c^5}\varepsilon^2 I^2$$

rotierender Balken




Doppelstern

i-Boo



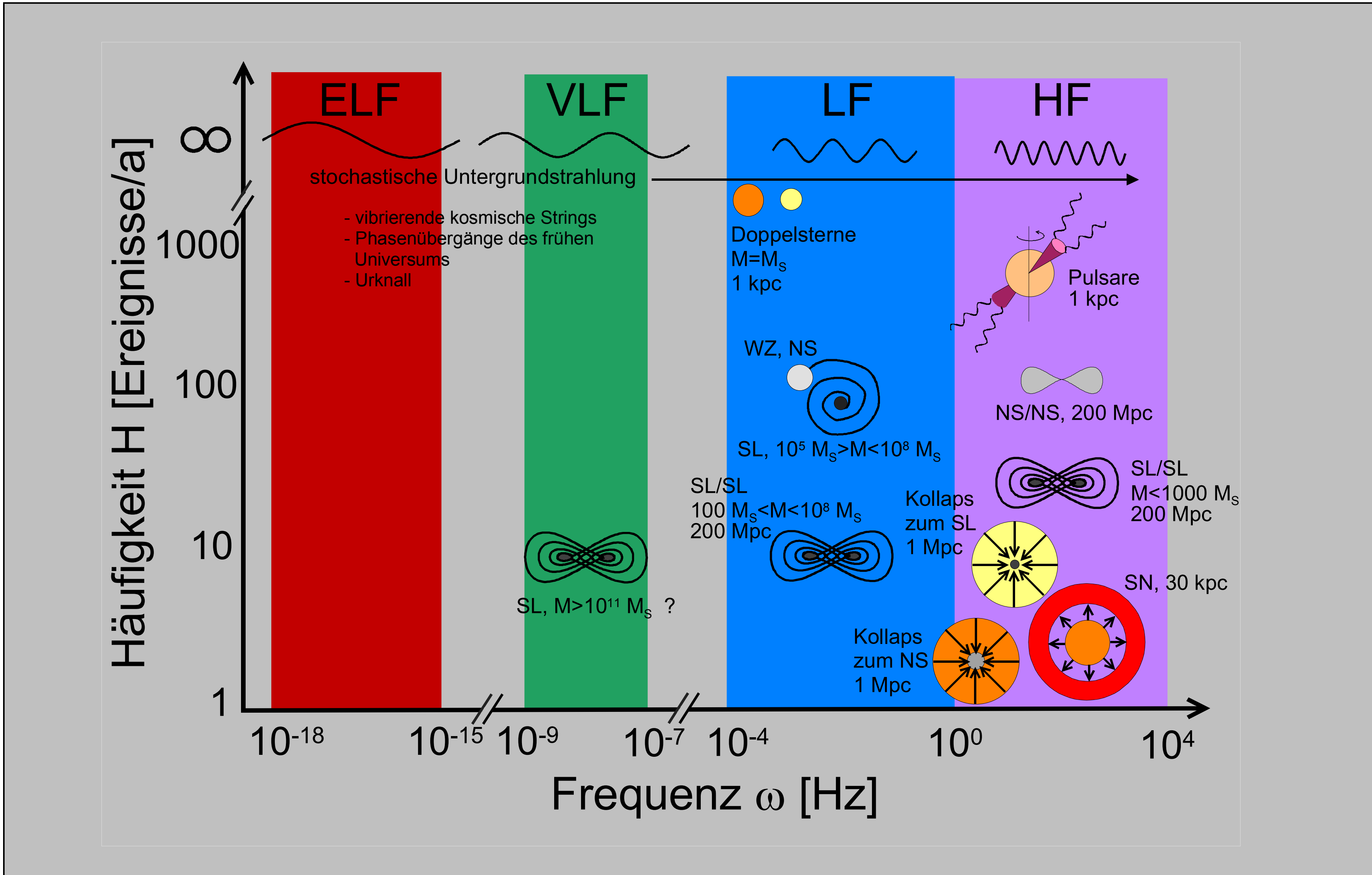

Supernova

**T=30 ms**
**P≈10-29 W**
**ΦGW ≈ P/L2=10-31 W/cm²**
**⇒ h=$10^{-34}$ m**

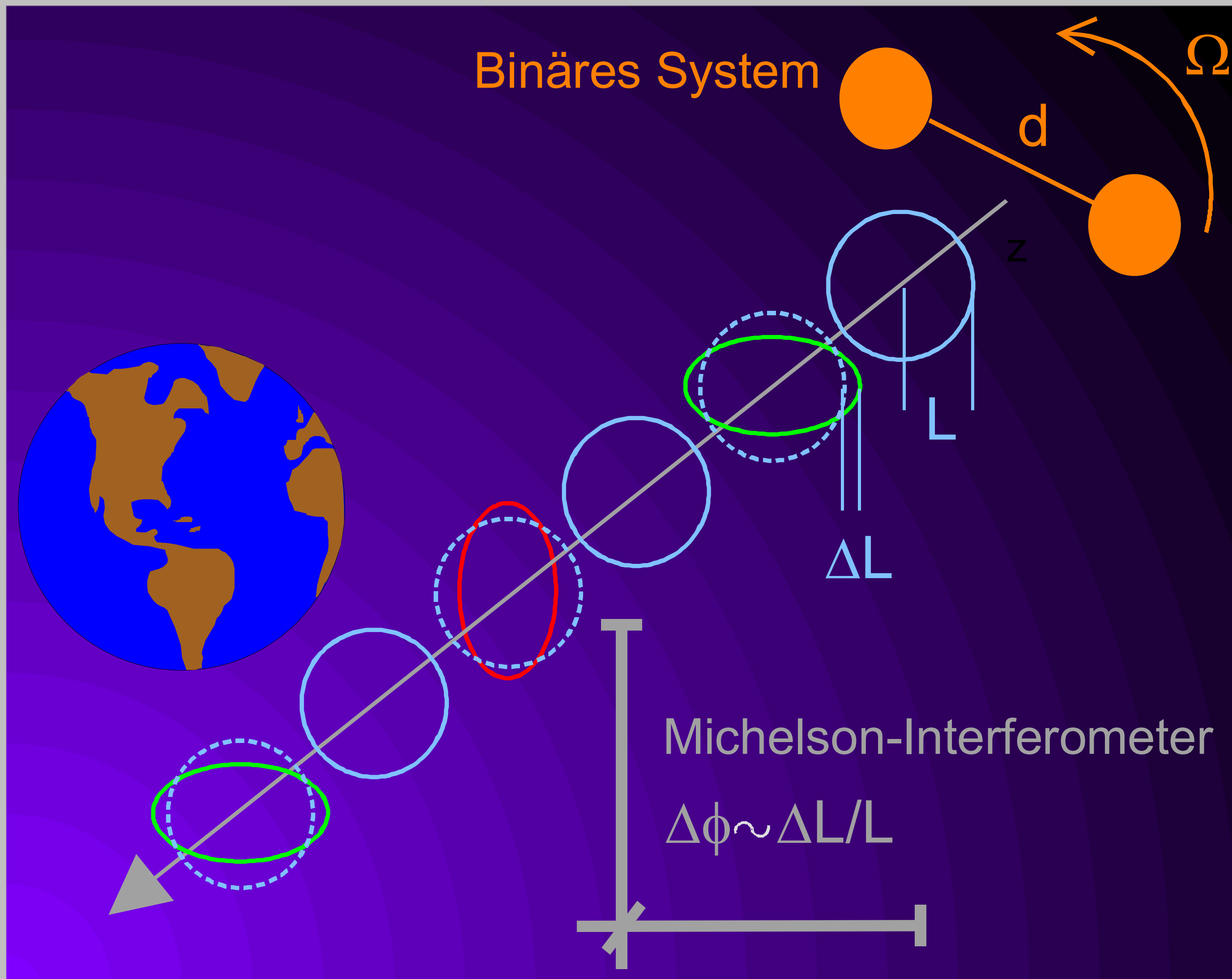
**T=0,268 d**
**P≈1023 W**
**ΦGW ≈ P/4πD2=10-13 W/cm²**
**⇒ h=$10^{-20}$ m**

**T=1 ms (Dauer 0,1 s!)**
**P≈1046 W**
**ΦGW ≈ P/4πD2=103 W/cm²**
**⇒ h=$10^{-19}$ m**

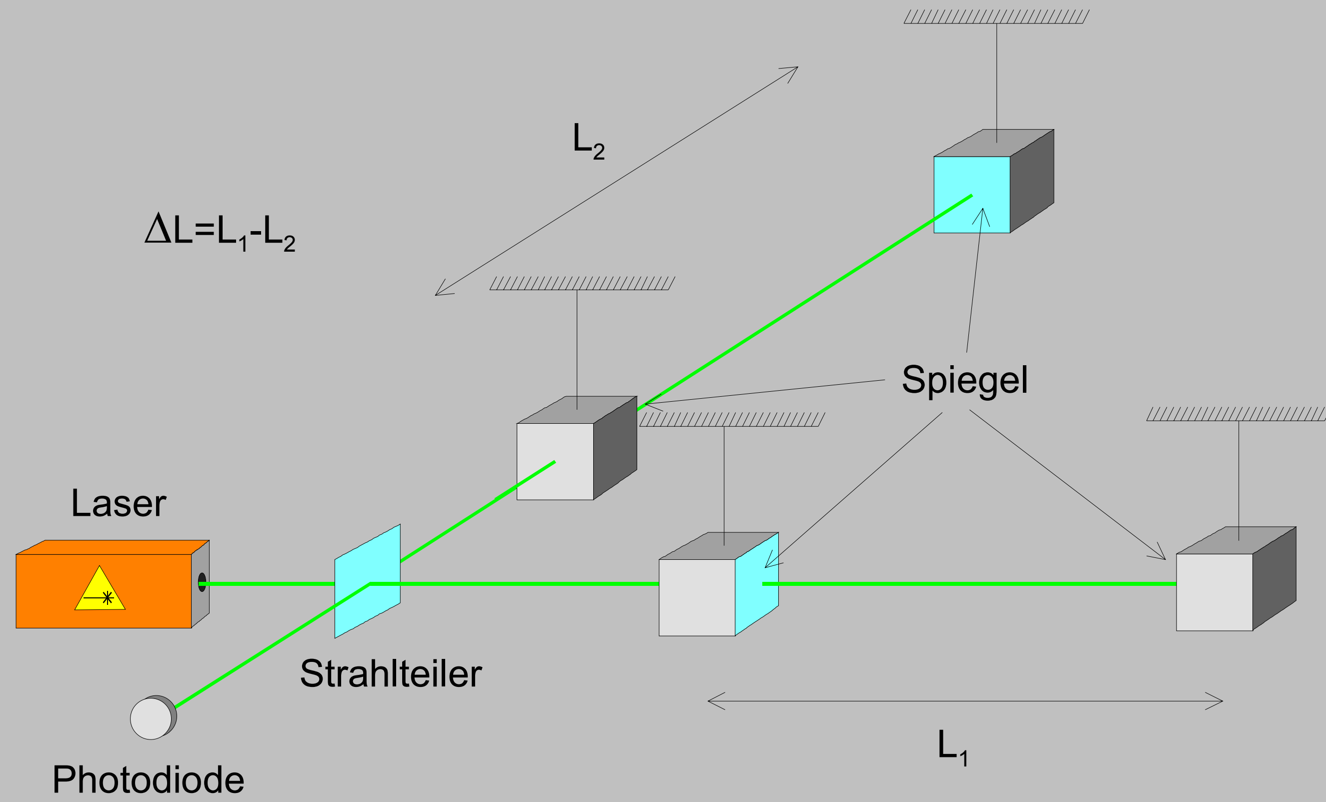
# Typische Vertreter, Frequenzen und Häufigkeiten



# Prinzip der interferometrischen Detektion



# Aufbau eines Laserinterferometers




$$\frac{\Delta L}{L} = F_{+} \cdot h_{+}(t) + F_{x} \cdot h_{x}(t) \equiv h(t)$$

# Erdgebundene Interferometer (LIGO)

Livingston (Louisiana)

Armlänge: **4 km**!

Hanford (Washington)

# Nachweisgrenze, Schrotrauschen

$$\Delta\Phi_{GW} = 2\pi \frac{B\Delta L}{\lambda_\gamma} = \frac{2\pi BL}{\lambda_\gamma} h$$

(Phasendifferenz durch GW)

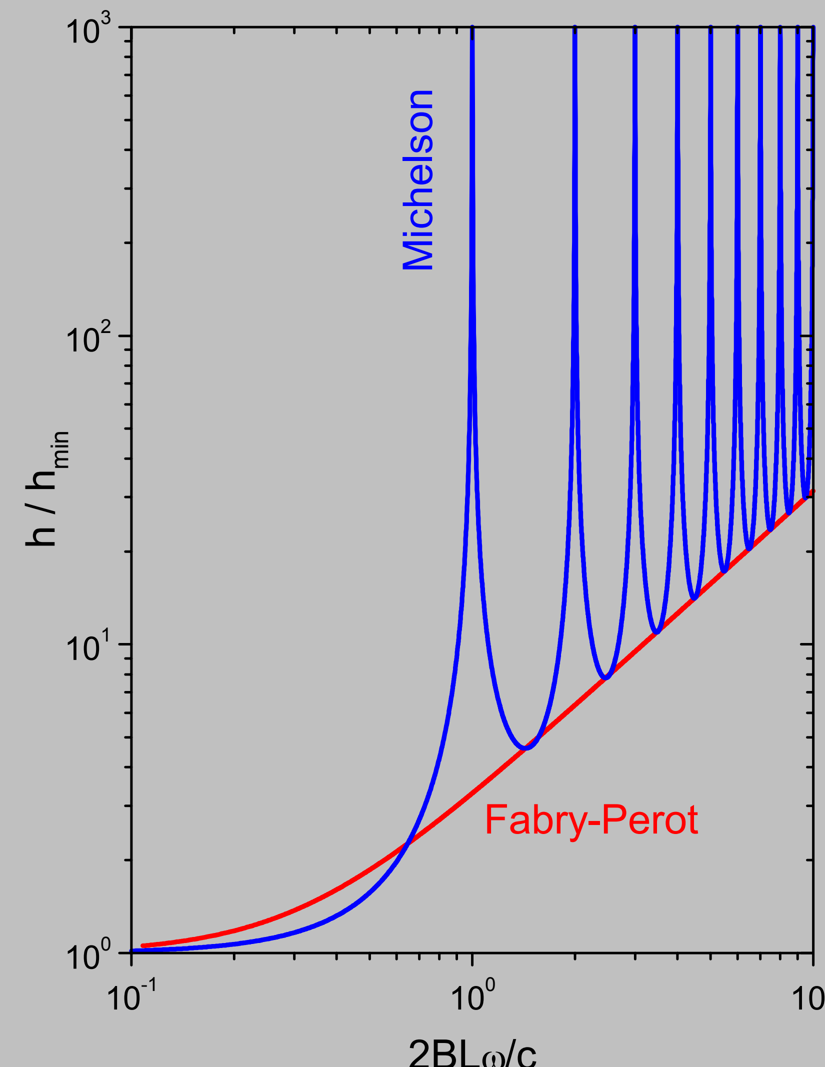
$$\Delta\Phi_{QM} \geq \frac{1}{2\Delta N} \approx \frac{1}{2\sqrt{N}}$$

(QM Phasenunschärfe)

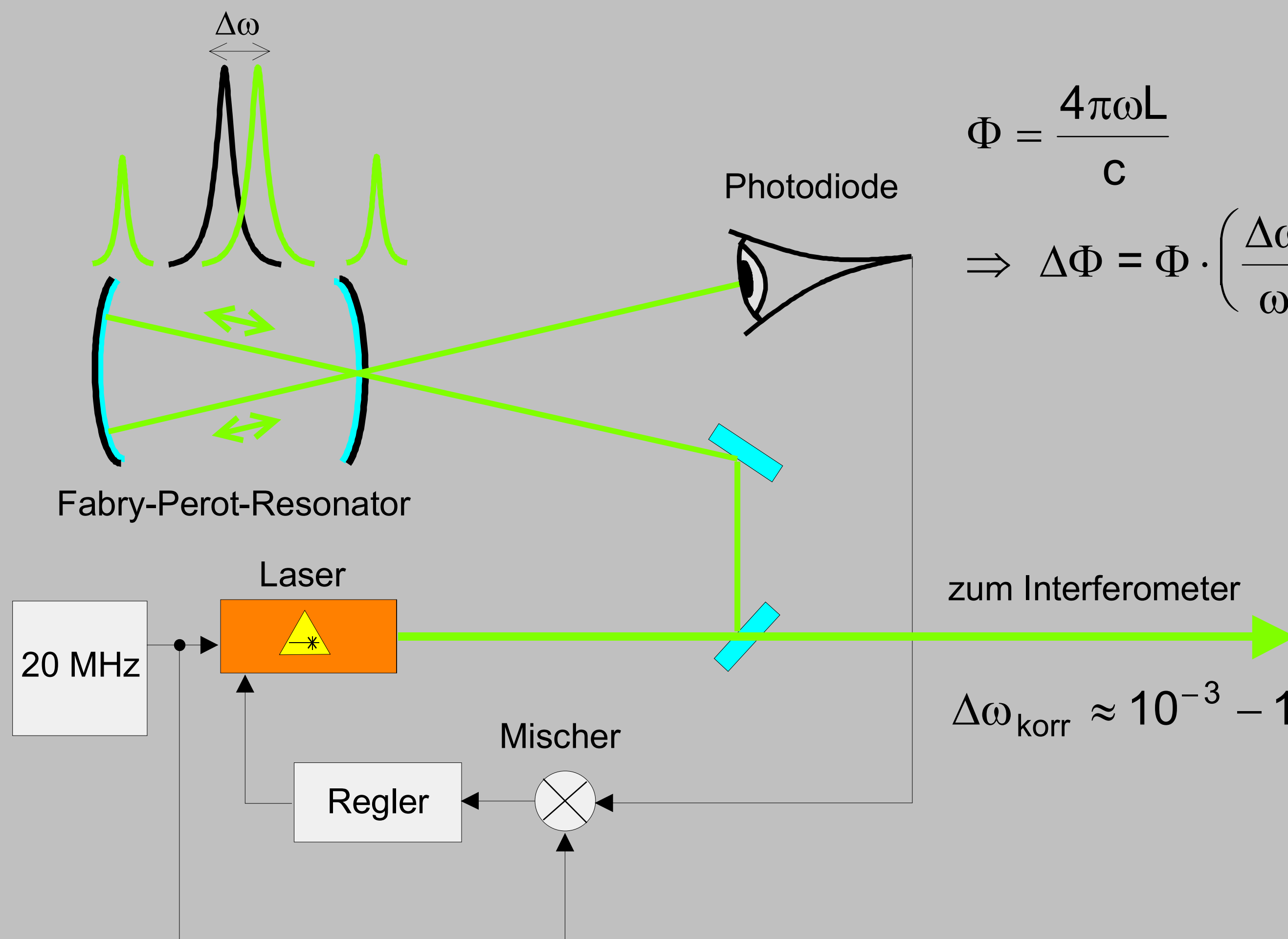
$N=P_\gamma \Delta t/\eta\omega_\gamma$ mit $\Delta t = BL/c$

$$\Rightarrow \quad h \geq \sqrt{\frac{c^2 \hbar \lambda_\gamma}{8\pi B^3 L^3 P_\gamma}} = h_{min}$$

Photonen-Schrotrauschen



# Frequenzstabilisierung (Pound, Drever, Hall)




$$\Phi = \frac{4\pi\omega L}{c}$$

$$\Rightarrow \Delta\Phi = \Phi \cdot \left(\frac{\Delta\omega}{\omega} + \frac{\Delta L}{L}\right)$$

$$\Delta\omega_{korr} \approx 10^{-3} - 10^{-4}\,\Delta\omega$$

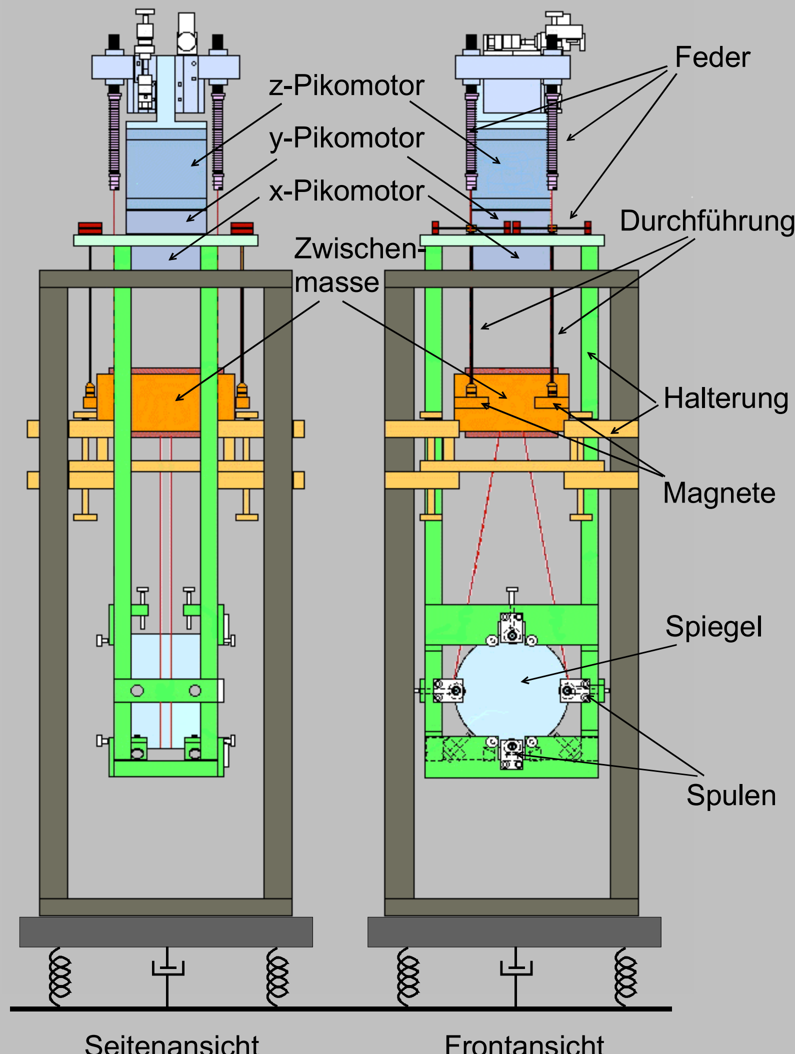
$$I \approx E_0^2 E_S \sqrt{R} / (1 - R) \sin(2\Delta\omega\Delta\tau)$$

# Vakuumsystem

$$\Delta\Phi = (n_{Luft} - 1)\frac{2\pi L}{\lambda}$$

Basisdruck: p=$10^{-9}$ bar !

# Mechanische Dämpfung

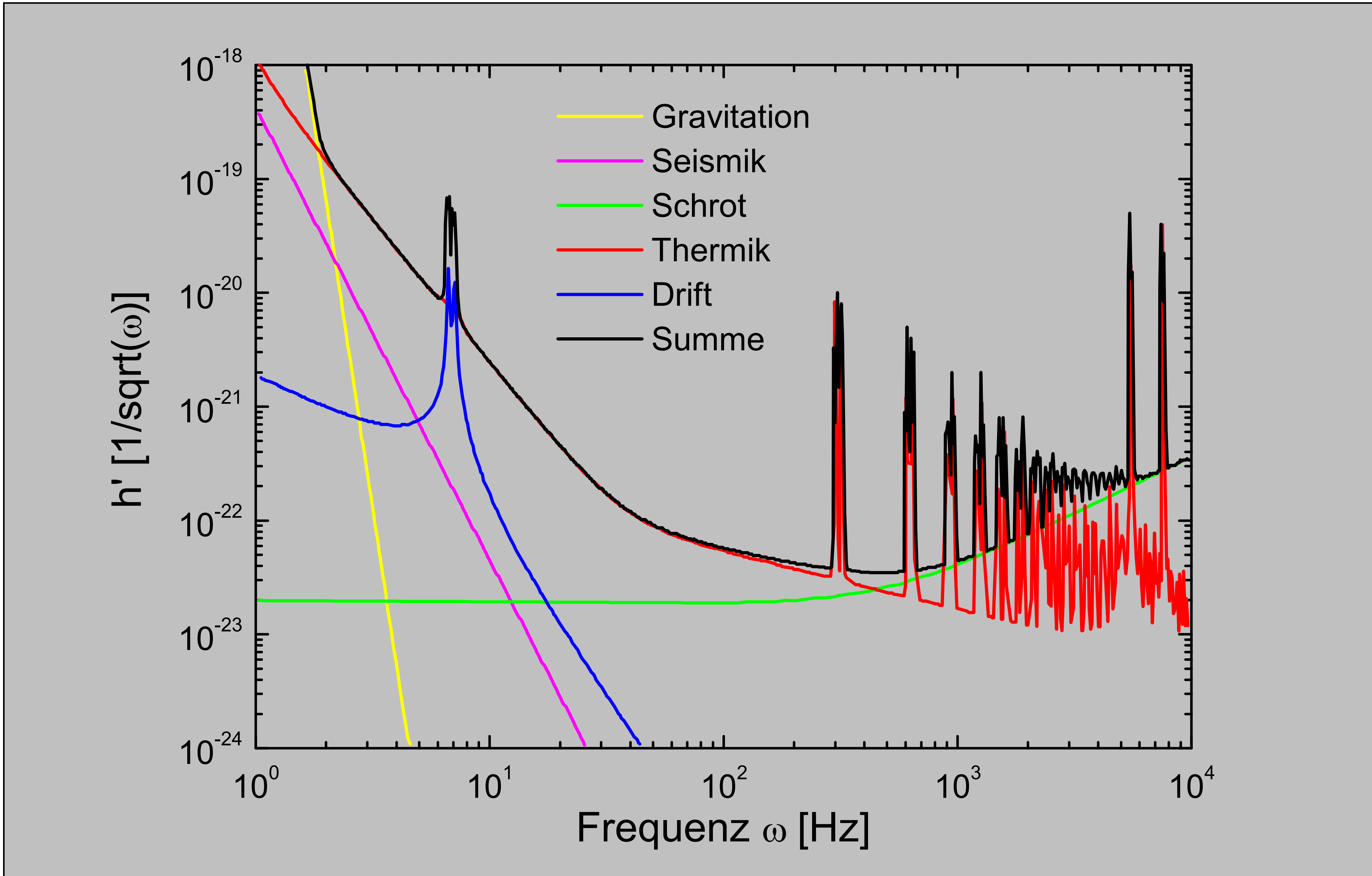



$SiO_2$-Spiegel

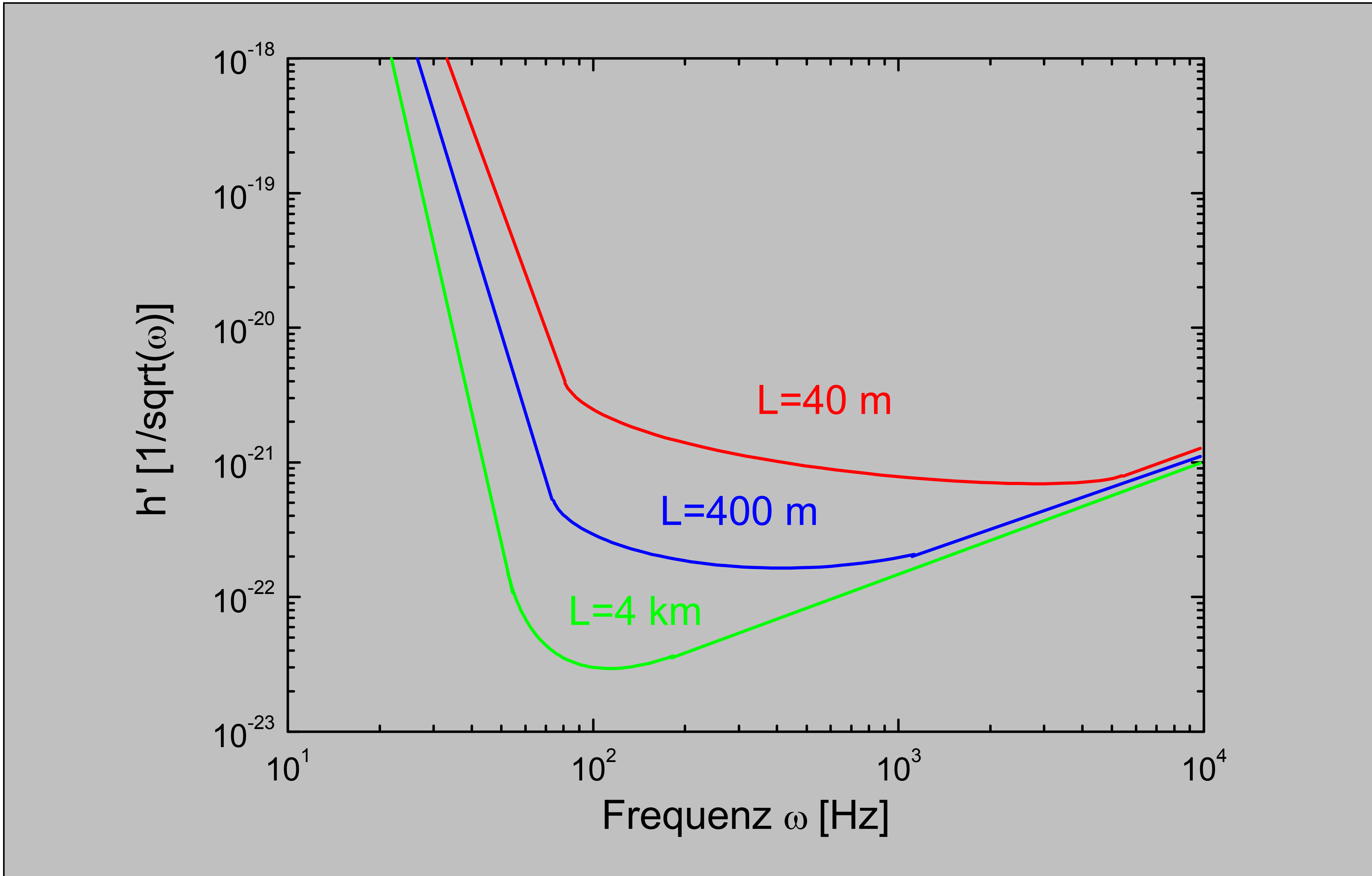
RMS<$\lambda$/600 !

$$Q = \frac{\omega}{\Delta\omega} = \frac{1}{2D} \approx 10^7$$

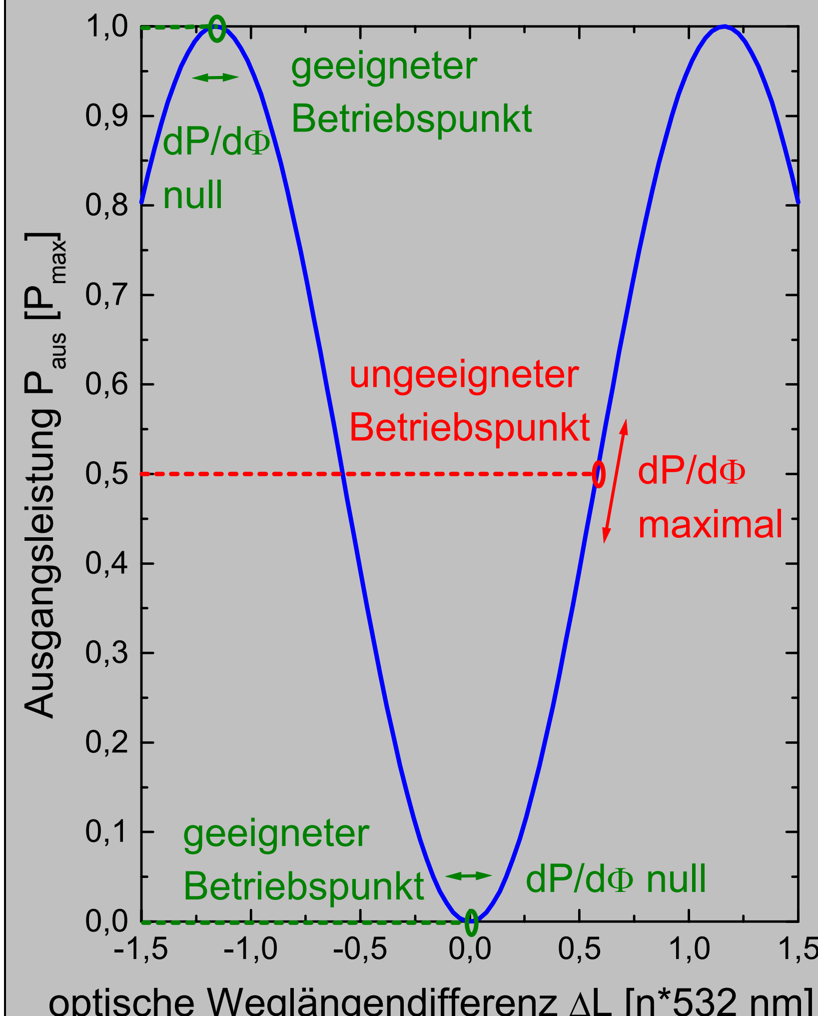
# Gesamtempfindlichkeit

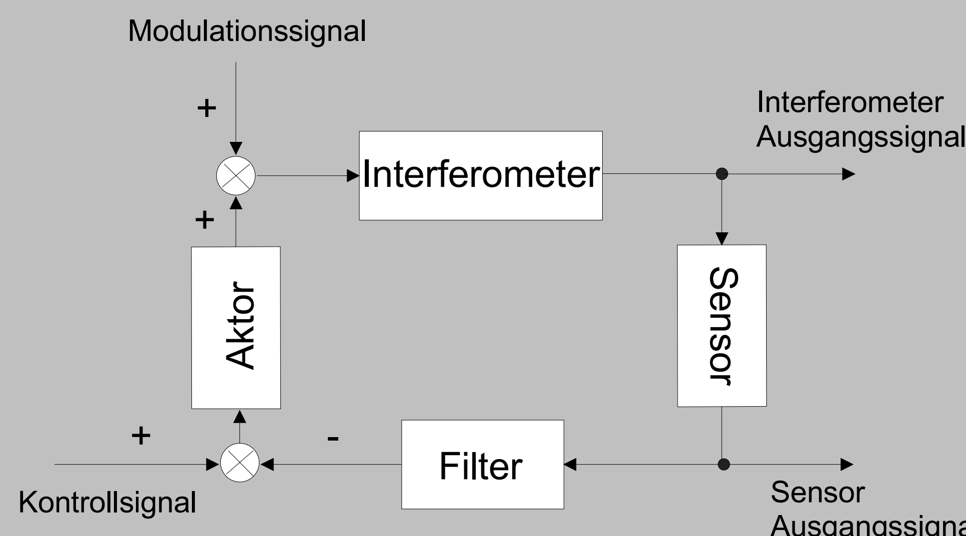


# Einfluß der Armlänge

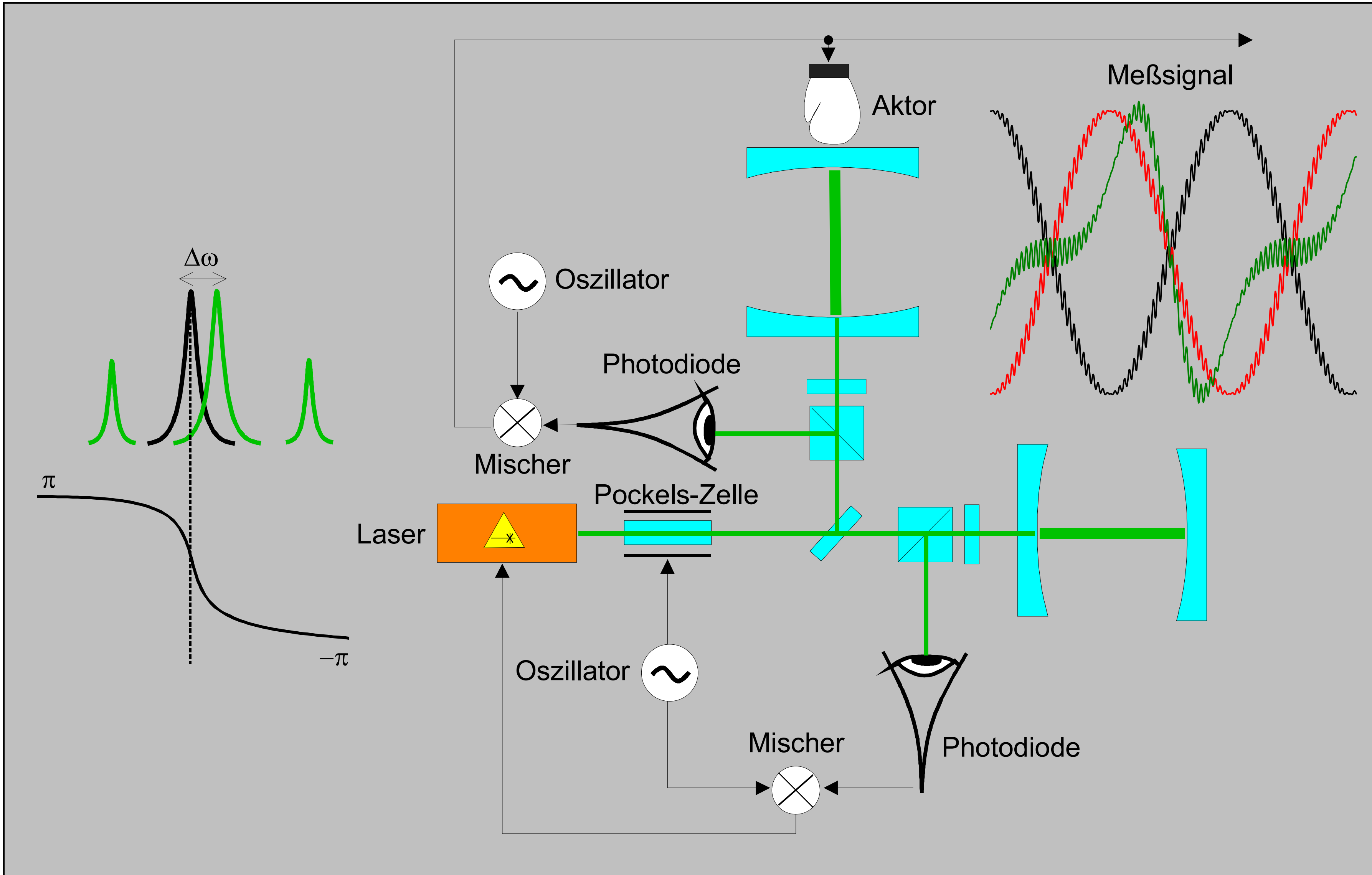


# Betrieb des Interferometers

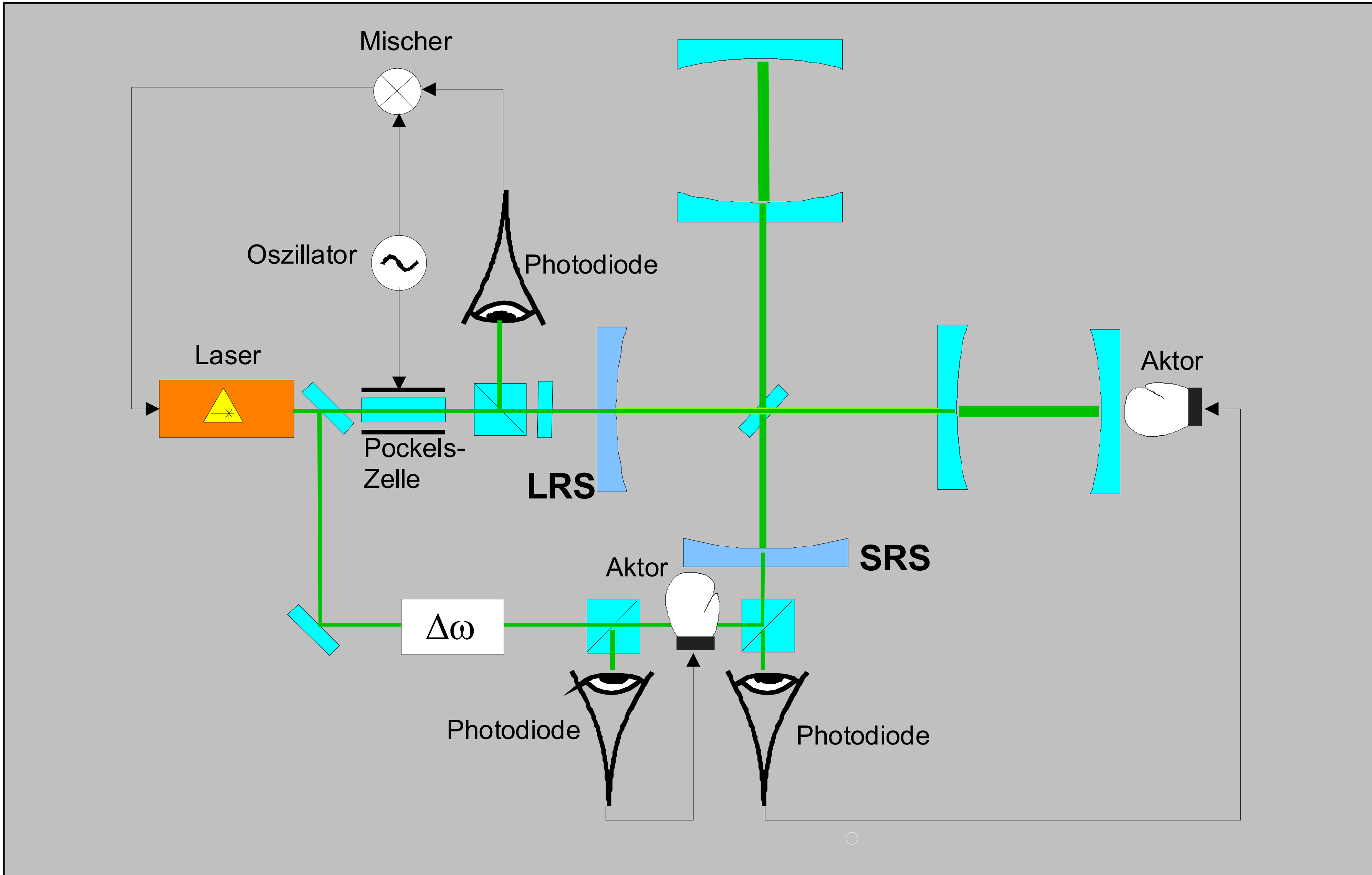


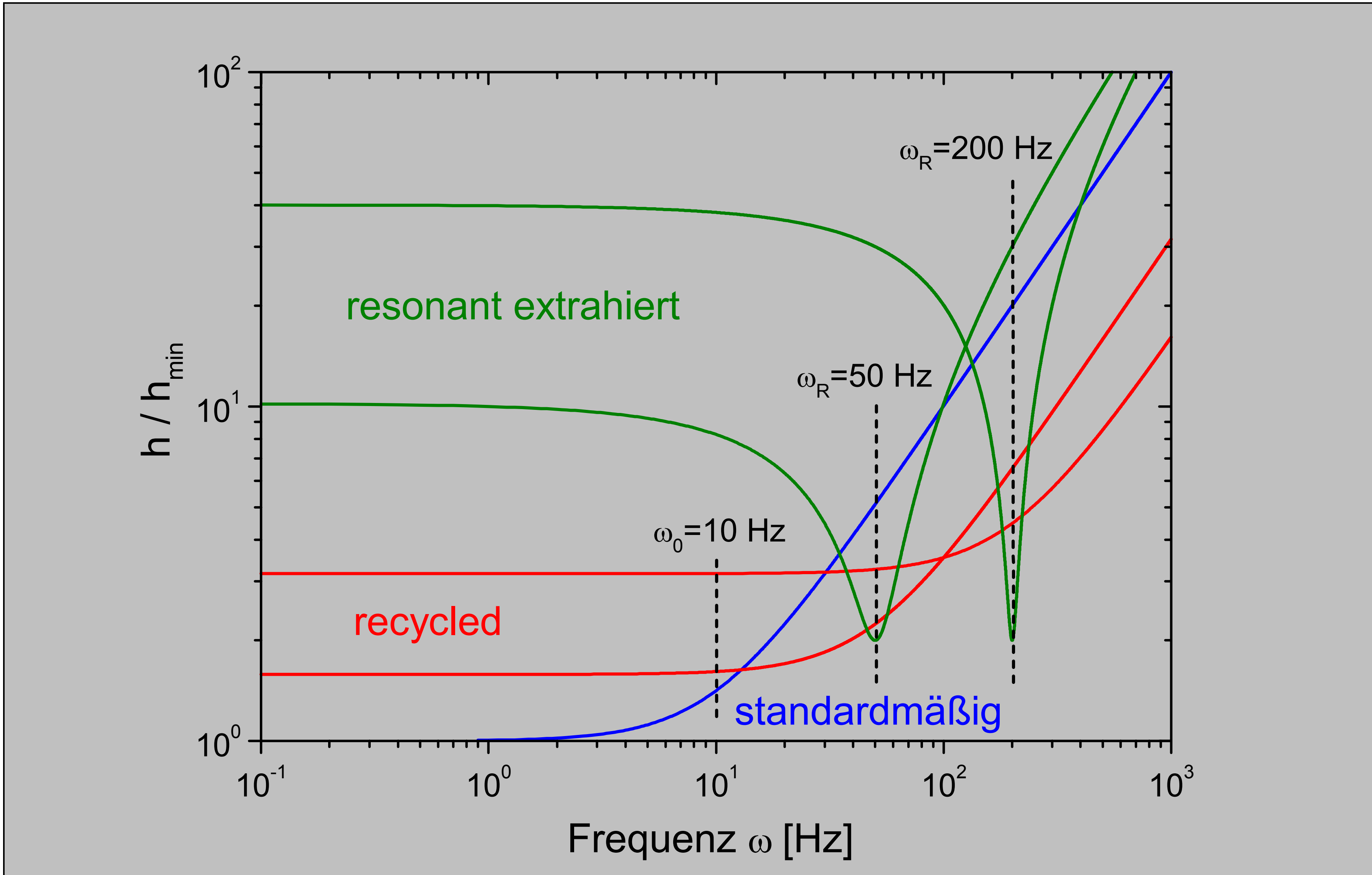
# Realisierung



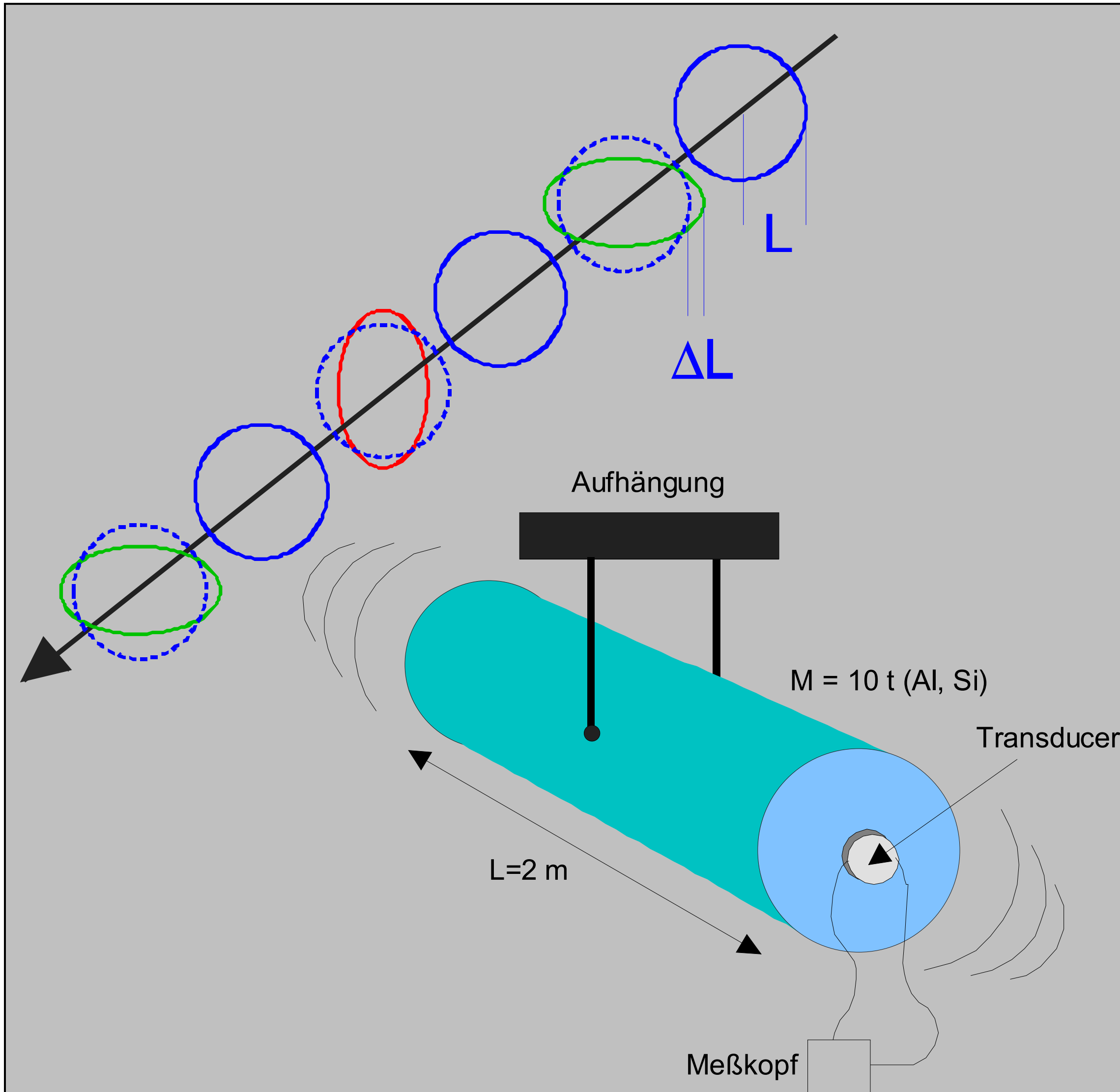
# Lichtrecycling und Seitenbandextraktion



# Empfindlichkeitssteigerung



# Prinzip einer resonanten Massenantenne




Fundamentalfrequenz

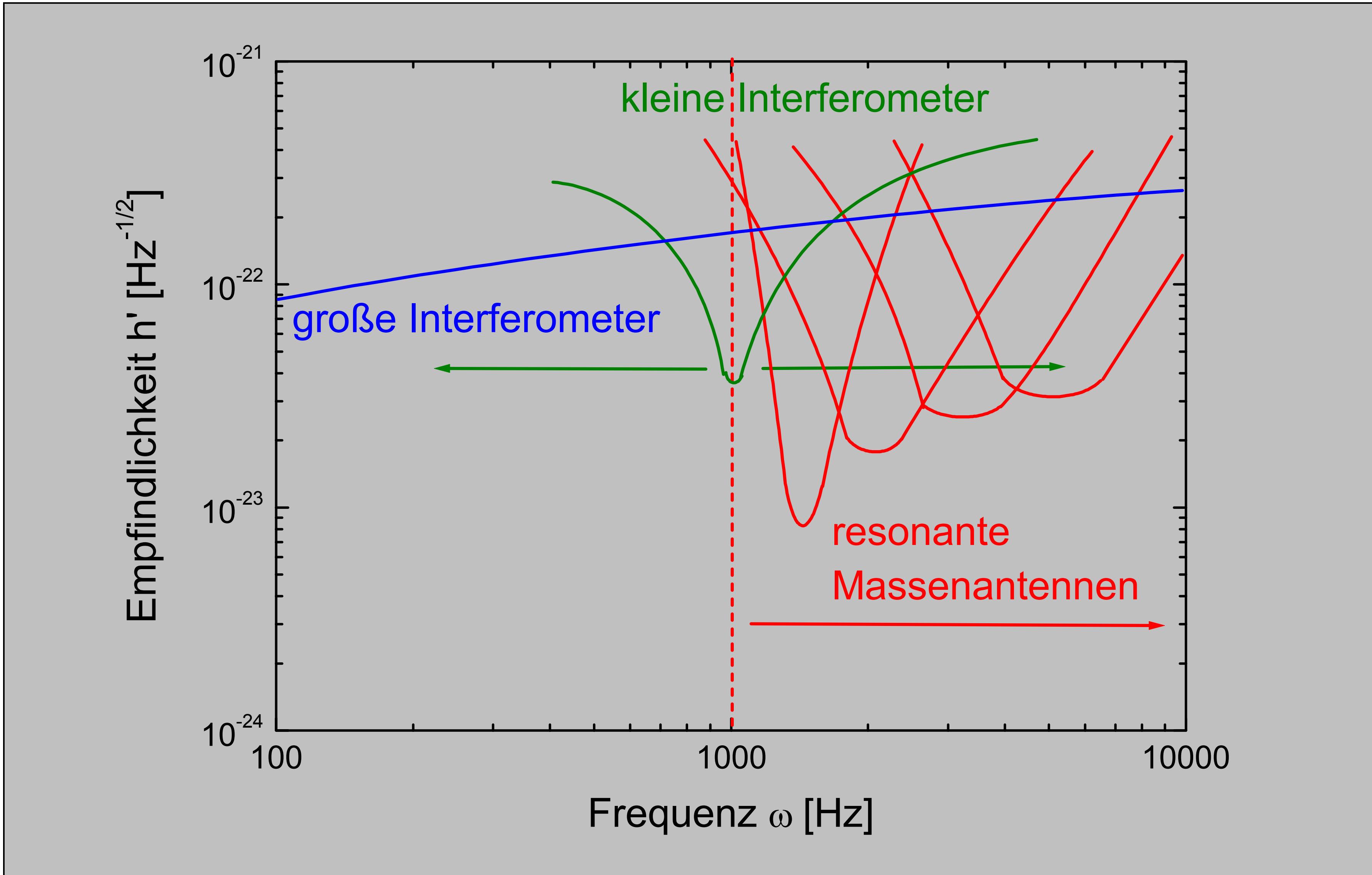
$$\omega_0 \approx \frac{v_s}{L} \approx 1\,\mathrm{kHz}$$

Empfindlichkeit

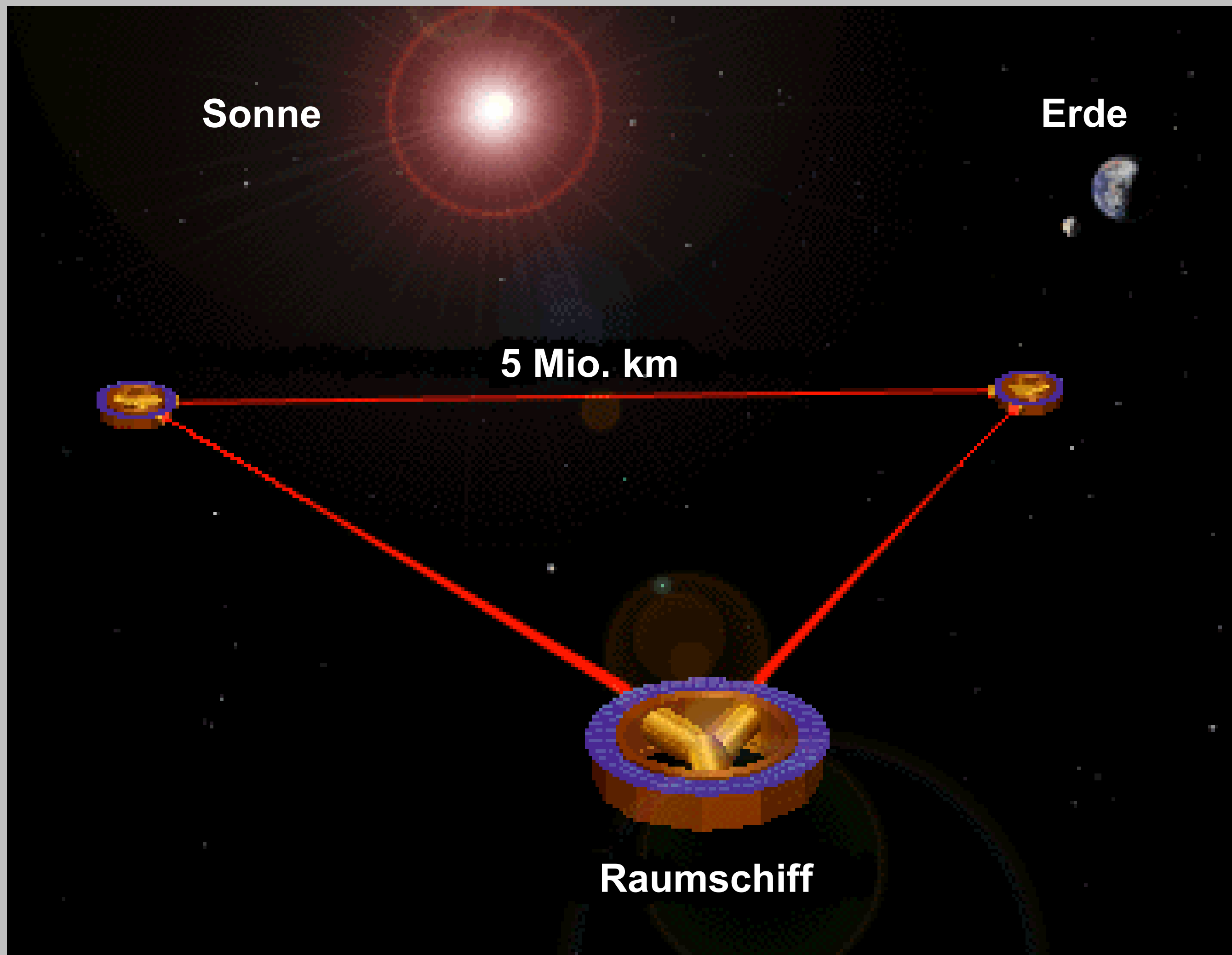
$$h_{min}(\tau) = \sqrt{\frac{2\pi^2 kT\gamma}{M\omega_0^4 L^2 \tau}}$$

$h_{min}(4\ \text{Monate}) \approx 10^{-22}$ !

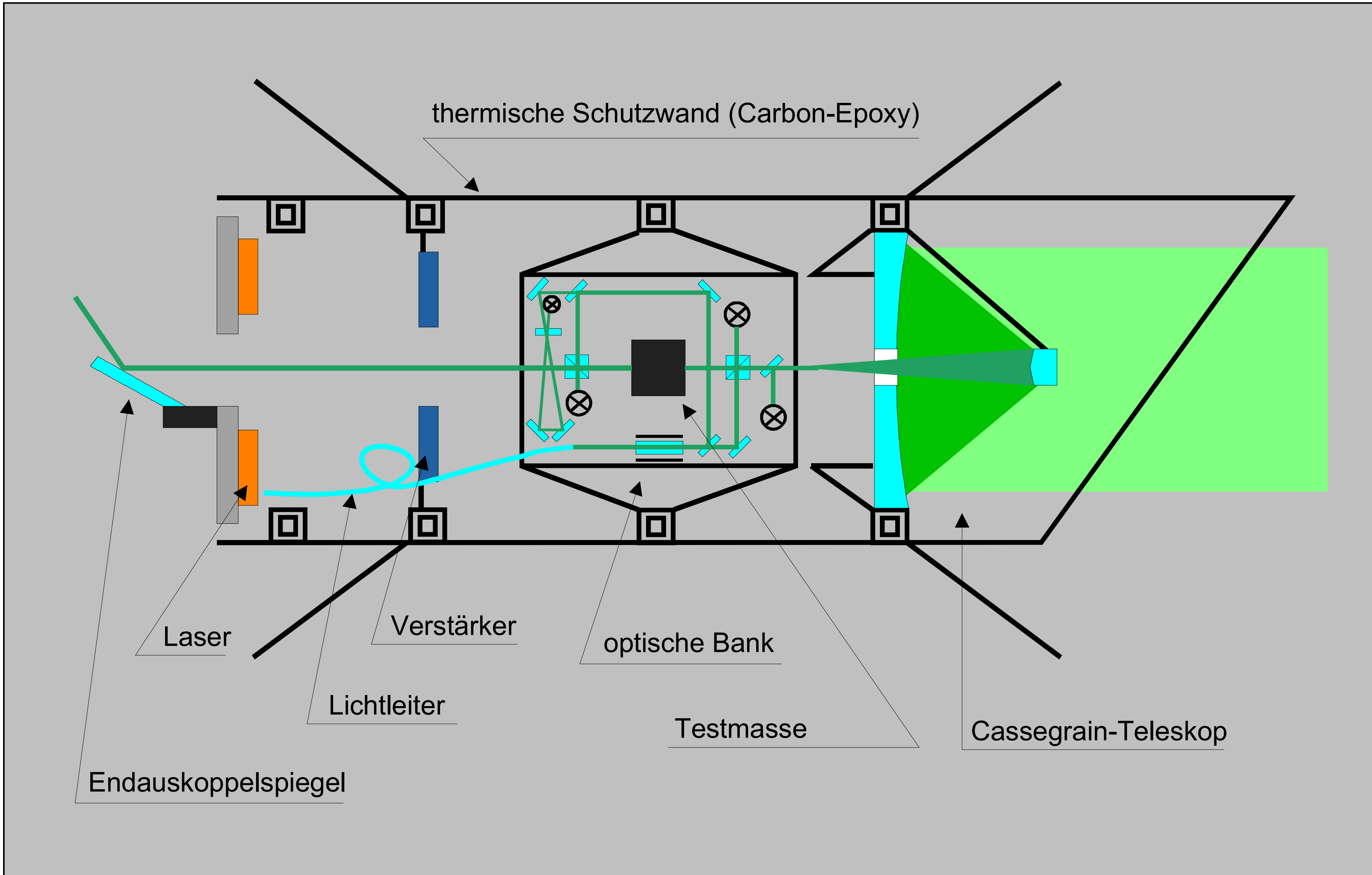
# Vergleich der Empfindlichkeiten



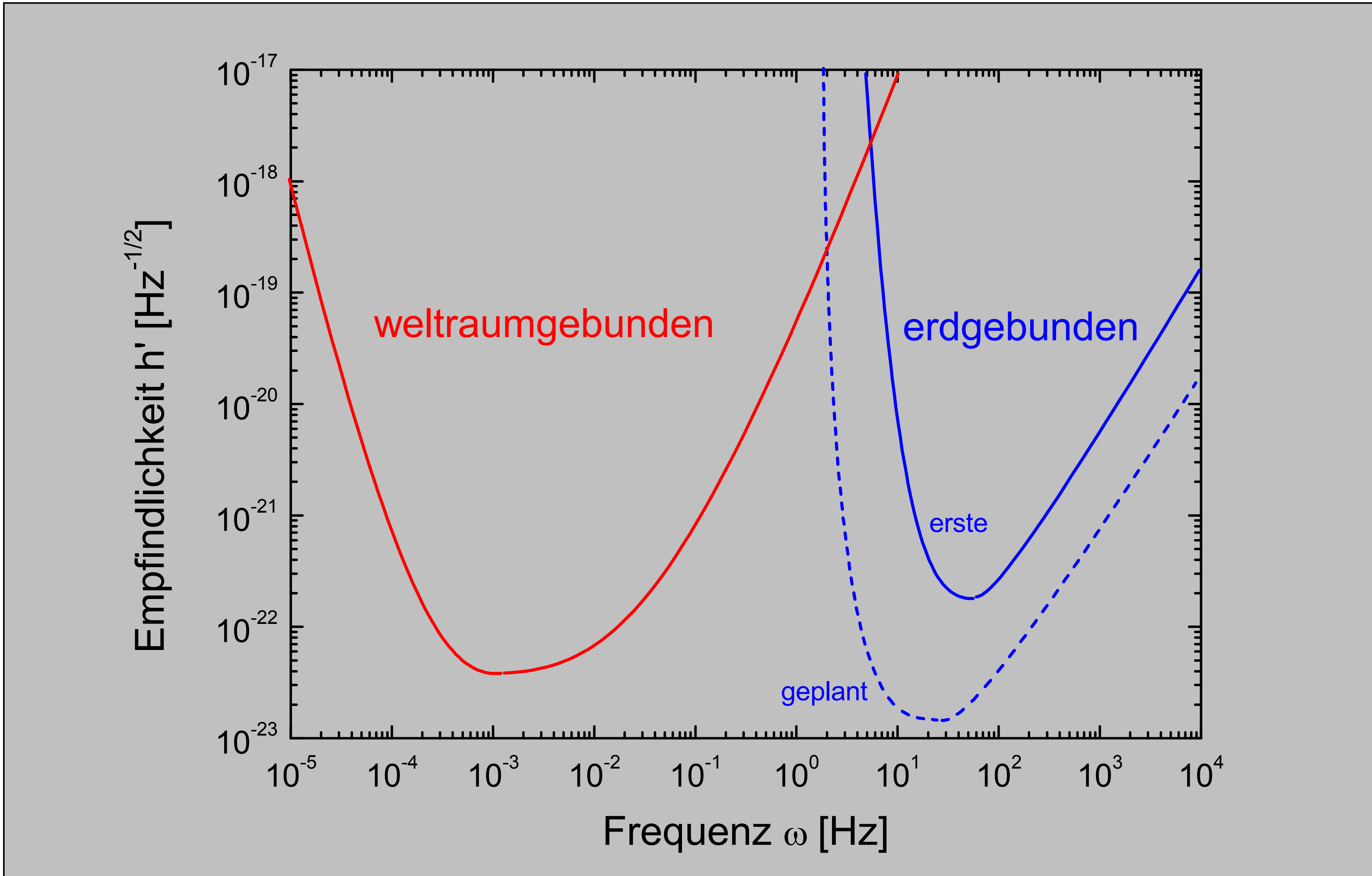
# Weltraumgebundene Interferometer (LISA)



# Optischer Aufbau



# Empfindlichkeit

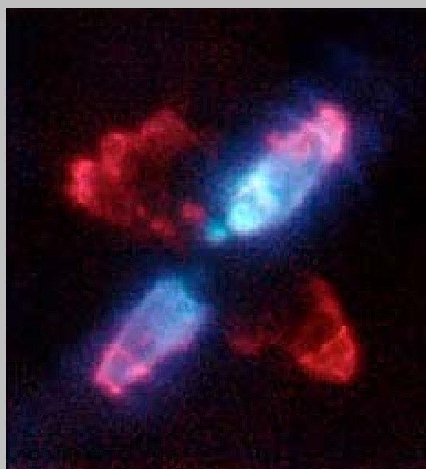


# Weltweite Großprojekte

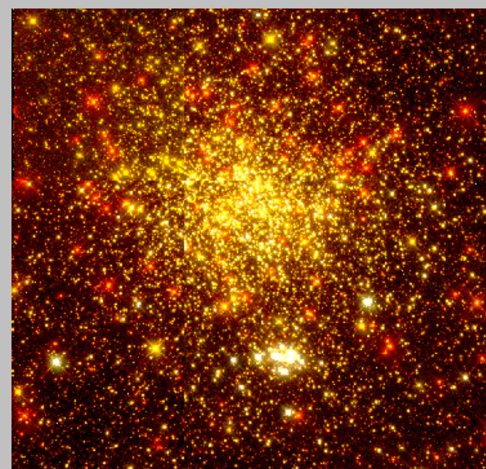
| **Projekt** | **Land** | **Art** | **Budget** [Mio US $] | **Start** | **Kooperation** |
|---|---|---|---|---|---|
| LIGO (**L**aser **I**nterferometric **G**ravitational Wave **O**bservatory) | USA | erdgeb., 4 km Armlänge (2 Stk.) | 80-100 | 2000 | VIRGO |
| VIRGO | Frankreich / Italien | erdgeb. 3 km Armlänge | 40 | 2000 | LIGO |
| GEO600 | Deutsch-land / Schottland | erdgeb. 600 m Armlänge | <20 | 2000 | VIRGO / LIGO ? |
| TAMA | Japan | erdgeb. 300 m Armlänge | <20 | 2000 | VIRGO / LIGO ? |
| LISA (**L**aser **I**nterferometric **S**pace **A**ntenna) | USA / Europa (NASA/ESA) | weltallgeb. 5 Mio. km Armlänge | >960 | 2008 | — |

# Zielsetzung

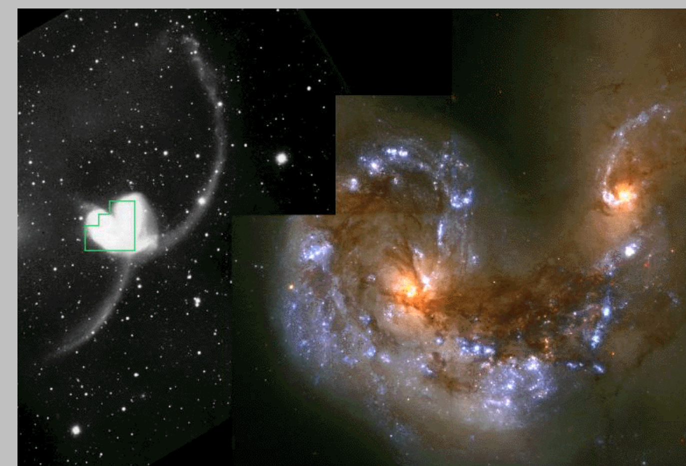
1) Experimenteller Nachweis von Gravitationswellen
2) Gravitationswellen-Astronomie



Supernovae

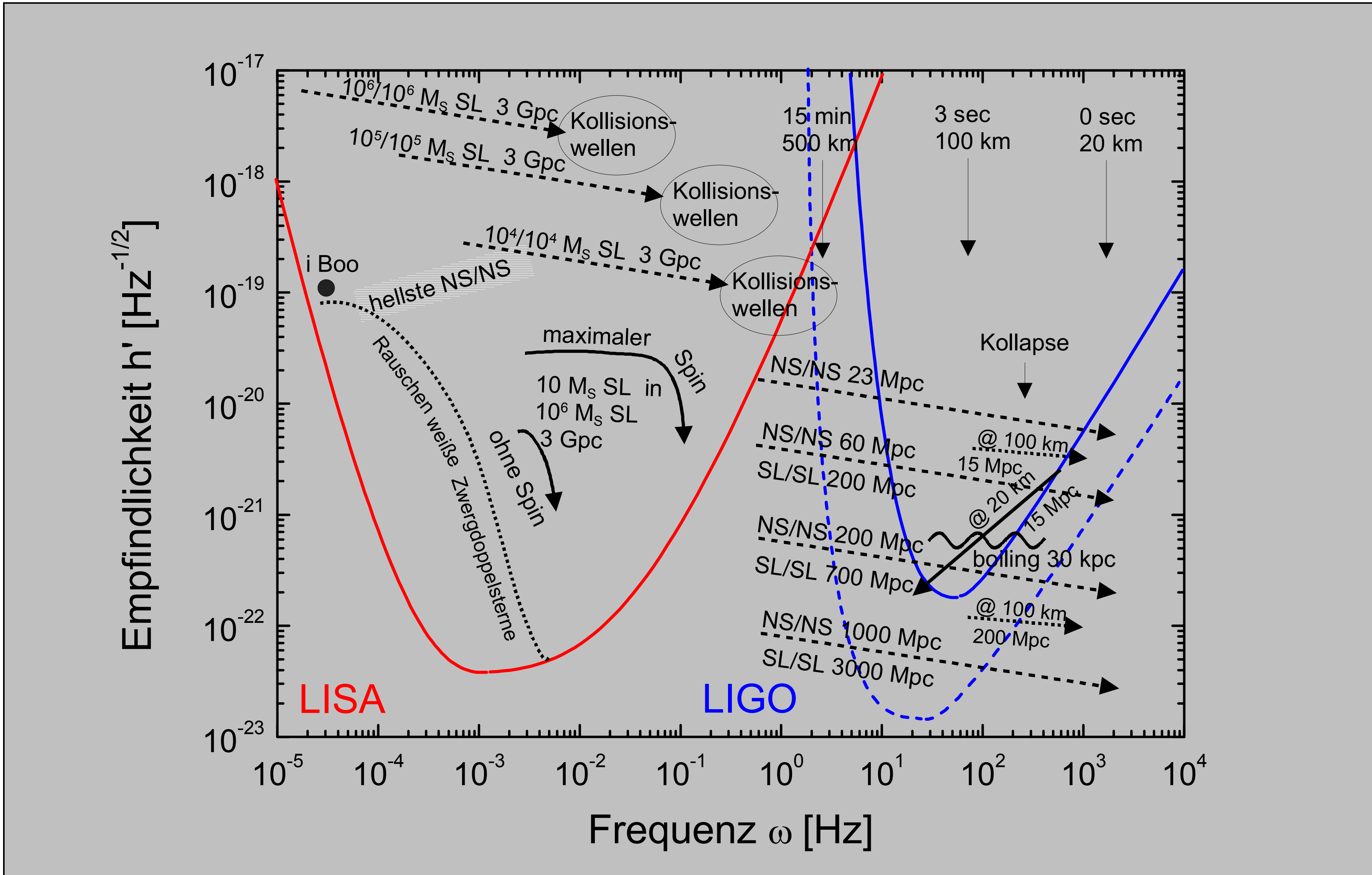


Sternkollisionen in Sternhaufen



Kollision supermassiver schwarzer Löcher in Galaxienzentren

# Gravitationswellenastronomie



# Zusammenfassung

- Indirekte experimentelle Evidenz für Gravitationswellen
- Große Vielzahl astronomischer Quellen

  $\Rightarrow$ neuer Zweig der Astronomie
- Experimenteller Nachweis mit Laser-Interferometern
- Vorteile: große Bandbreite
  hohe Empfindlichkeit
  niedrige Frequenzen zugänglich
- Weltweit 5 Großprojekte
- Erste Resultate im Jahr 2000 erwartet